Introduction to Game Theory:

Game Trees

Version 10/29/17

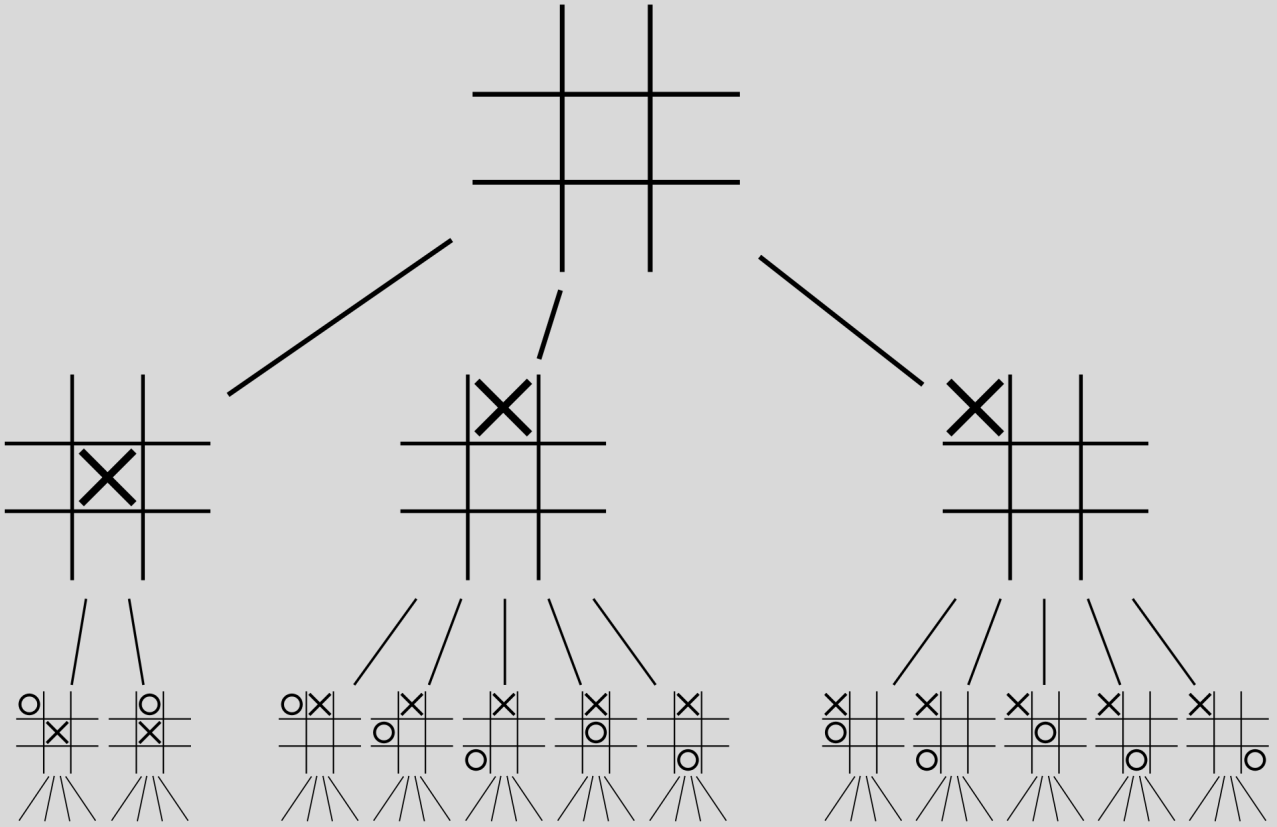
A Second “Non-Cooperative” Game Model

The game tree is a model specifying which player moves first and what possible **moves** are available to that player; which player moves next and what moves are available to that player; and so on

Also specified is the **information** available to each player at each of the contingencies under which that player might need to choose a move

This information is made up of any information the player has about previous moves made by other players (and by himself, if we want to allow for limited memory) and about previous moves by Nature (i.e., chance moves)

Tic-Tac-Toe Revisited (Symmetrized Game Tree)

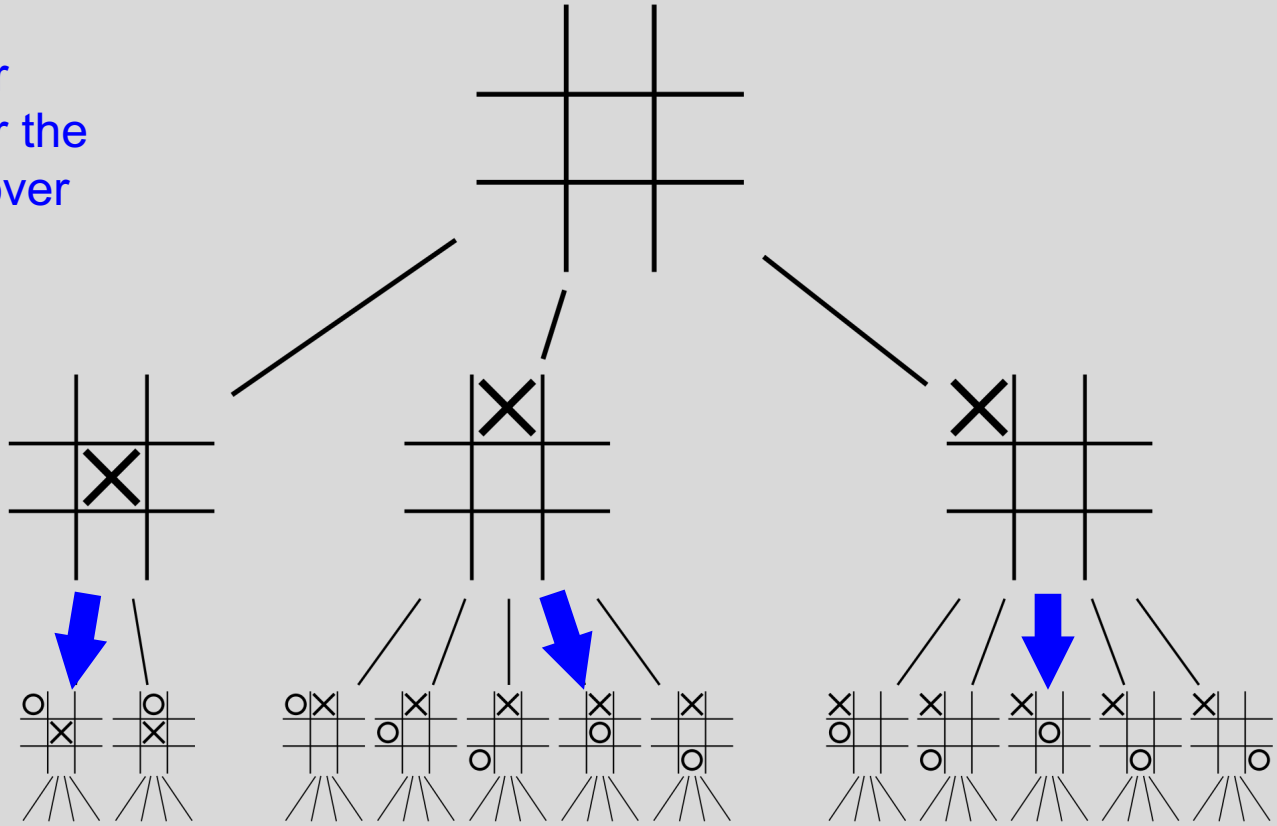


http://commons.wikimedia.org/wiki/File:Tic-tac-toe-game-tree.svg

Definition of a Strategy

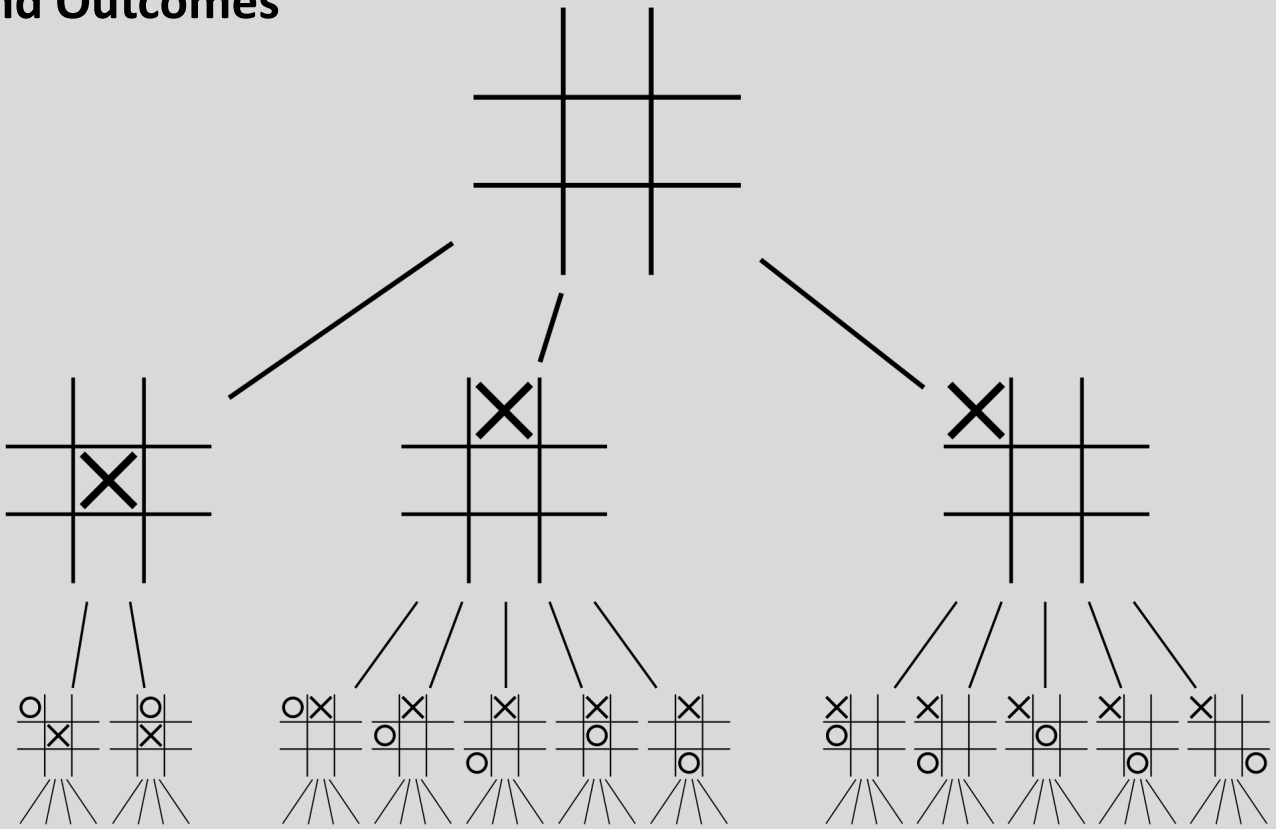
A **strategy** for a player specifies, at each node (later: information set) that belongs to that player, a particular move

A particular strategy for the second mover



http://commons.wikimedia.org/wiki/File:Tic-tac-toe-game-tree.svg

Endpoints and Outcomes



An endpoint

An outcome

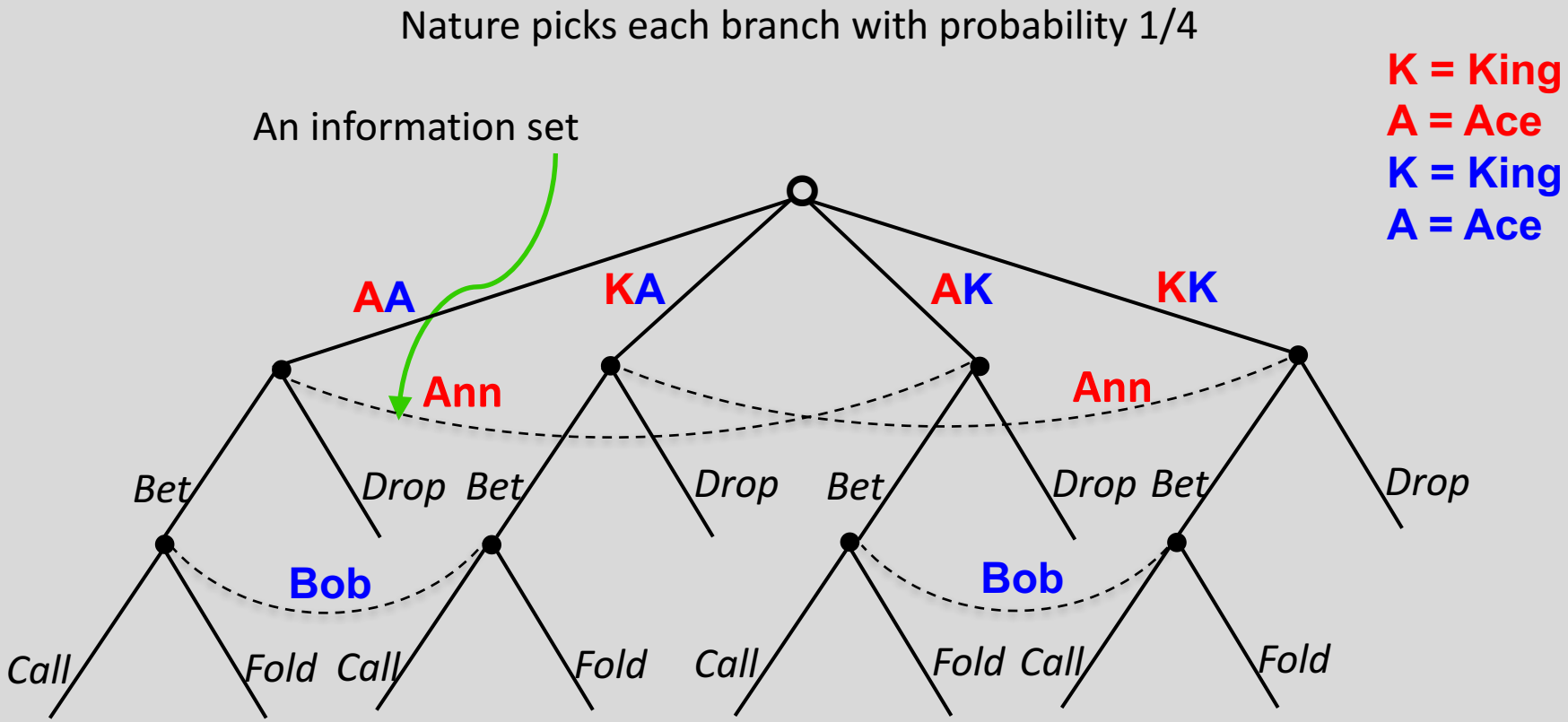
Ann's payoff Bob's payoff

What if payoffs are not 'transparent' to the players? Soon!

http://commons.wikimedia.org/wiki/File:Tic-tac-toe-game-tree.svg

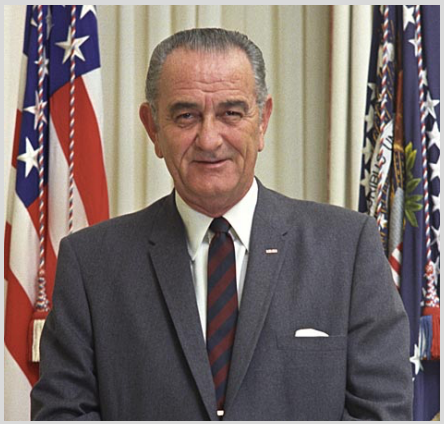
Simplified Poker

A move (or moves) by Nature can be used to expand the applicability of the game-tree model



Case Study: A Political 'Game'

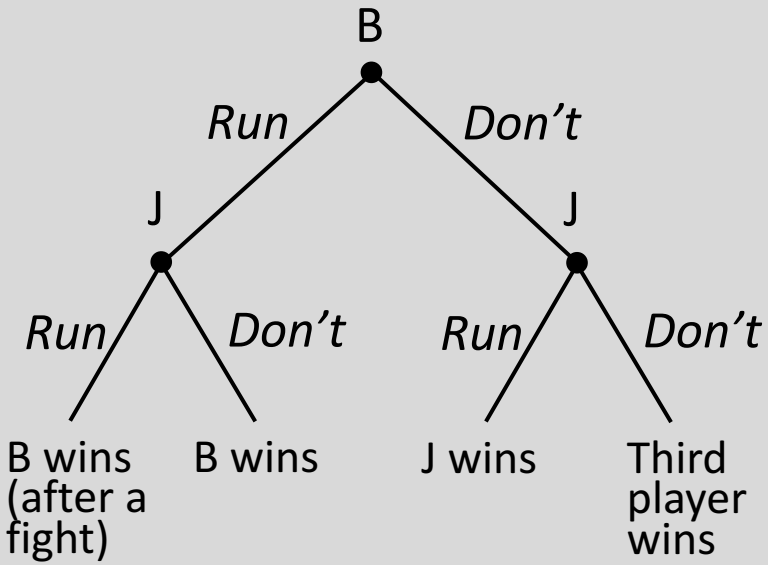
Lyndon Johnson was in Houston on February 23 [1937] … when he suddenly saw, on a park bench, a copy of the *Houston Post* with the banner headline: CONGRESSMAN JAMES P. BUCHANAN OF BRENHAM DIES. He knew at once, he was to recall, that "this was my chance…."

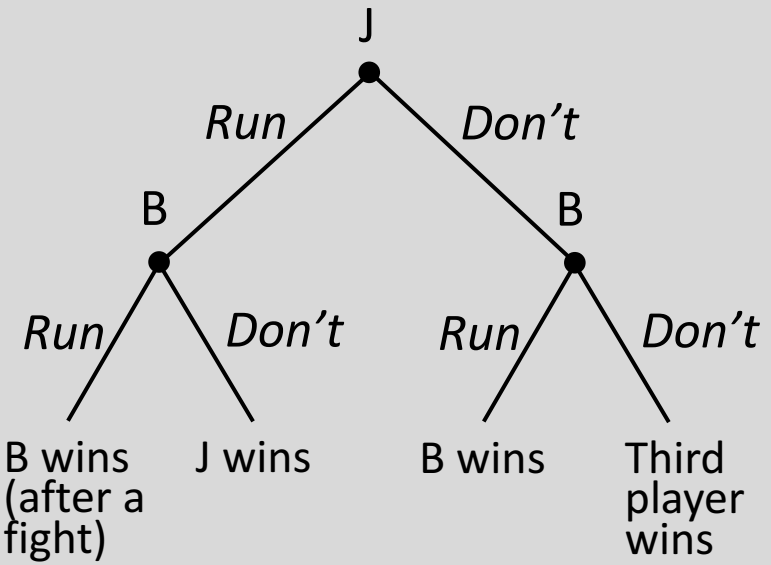


A strategy, money, an organization --- these would give this unknown candidate [Johnson] a slim chance of victory against every opponent but one. Against that one opponent, nothing could give him a chance. Nothing could offset the sentimental appeal of a vote for Old Buck's [Buchanan's] widow.... And it began to look as if she was going to run…. So Lyndon Johnson went … and asked his father's advice. Sam Johnson [Lyndon's father] didn't even have to think before giving it. Recalls Lyndon's brother: "Lyndon started saying he was thinking of waiting to see what she [Mrs. Buchanan] does, and Daddy says, "Goddammit, Lyndon, you never learn anything about politics." Lyndon says, "What do you mean?"

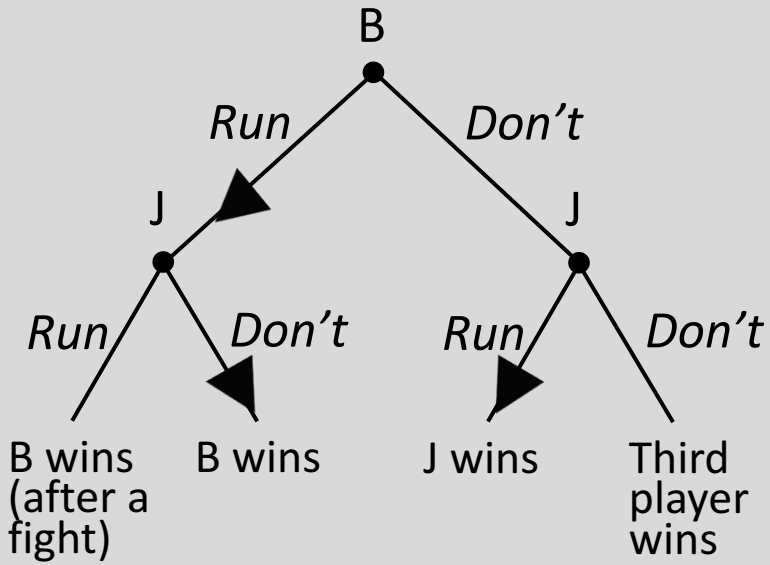
 From: *The Years of Lyndon Johnson: The Path to Power*, by Robert Caro, Vintage, 1990, pp.389-399; photo from https://media1.britannica.com/eb-media/18/126018-004-EC84F13F.jpg

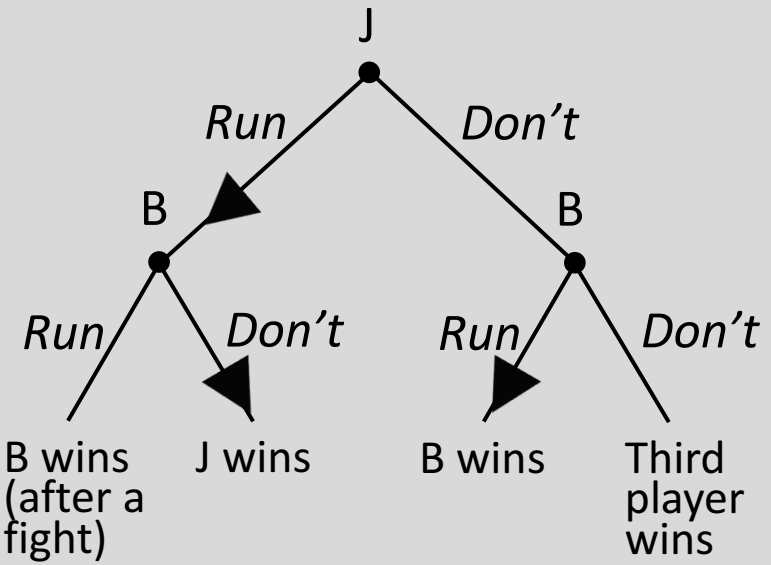
Case Study Cont'd: Two Game Trees





Case Study Cont'd: Backward Induction

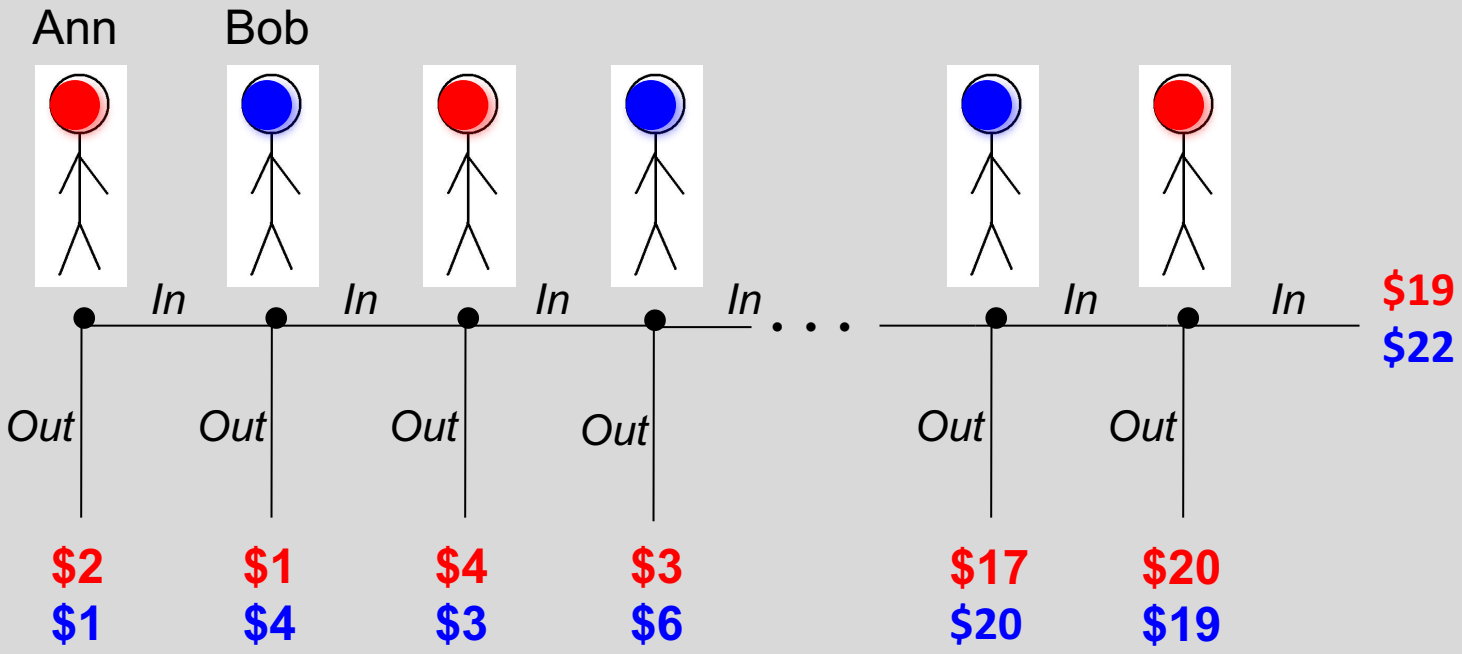




The Backward-Induction Algorithm

- Start at final decision nodes --- i.e., at decision nodes that lead only to terminal nodes of the tree
- Select the best choice for the relevant player at each of these nodes and replace the nodes with the resulting payoffs (to all the players)
- Repeat this process on the pruned tree
- The process ends when it reaches the root of the tree

The Centipede Game



What does the backward-induction algorithm yield in this tree?

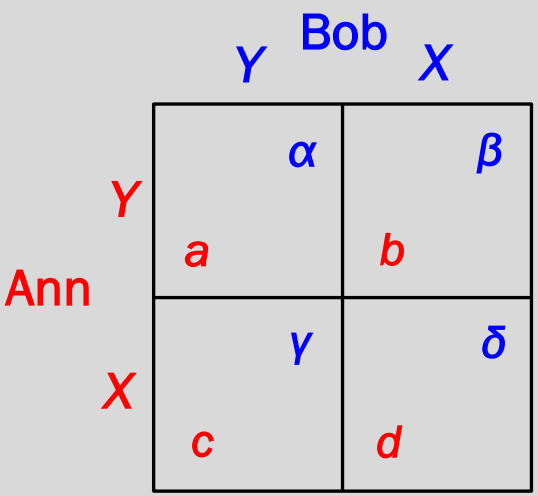
Rosenthal, R., "Games of Perfect Information, Predatory Pricing, and the Chain Store," *Journal of Economic Theory*, 1980, 25, 92-100; McKelvey, R., and T. Palfrey, "An Experimental Study of the Centipede Game," 1992, *Econometrica*, 60, 803-836; http://commons.wikimedia.org/wiki/File:Stick_Figure.jpg

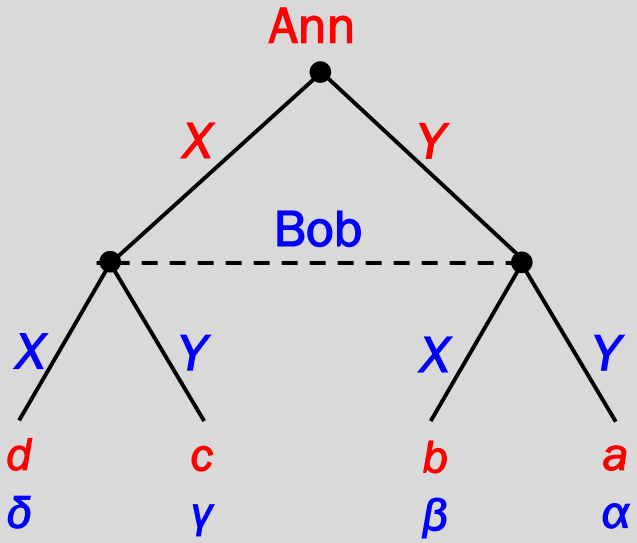
Some Game Tree Terminology and Facts

A game tree has **perfect** (respectively, **imperfect**) information if every information set is (respectively, is not) a singleton

The backward-induction algorithm applies to perfect-information trees

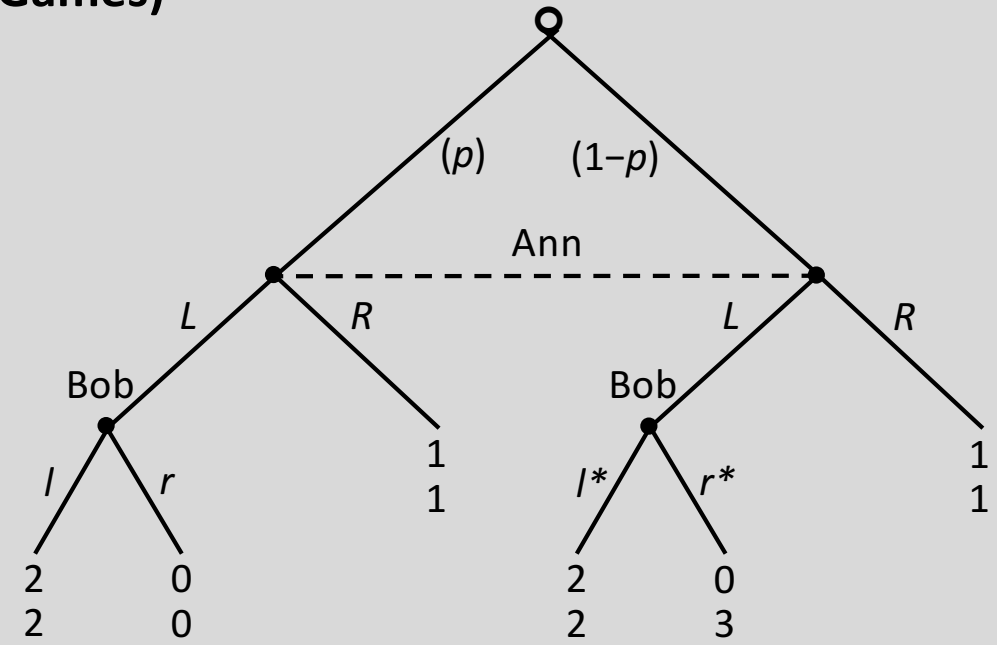
A game matrix can be viewed as a particular ('simultaneous-move') tree





(Advanced topic we will not pursue here: Every tree can be mapped to a (perhaps, big) matrix, so, what, at a deeper level, is the relationship between these two types of model?)

Games with Uncertainty About Payoffs (aka Bayesian or Incomplete-Information Games)

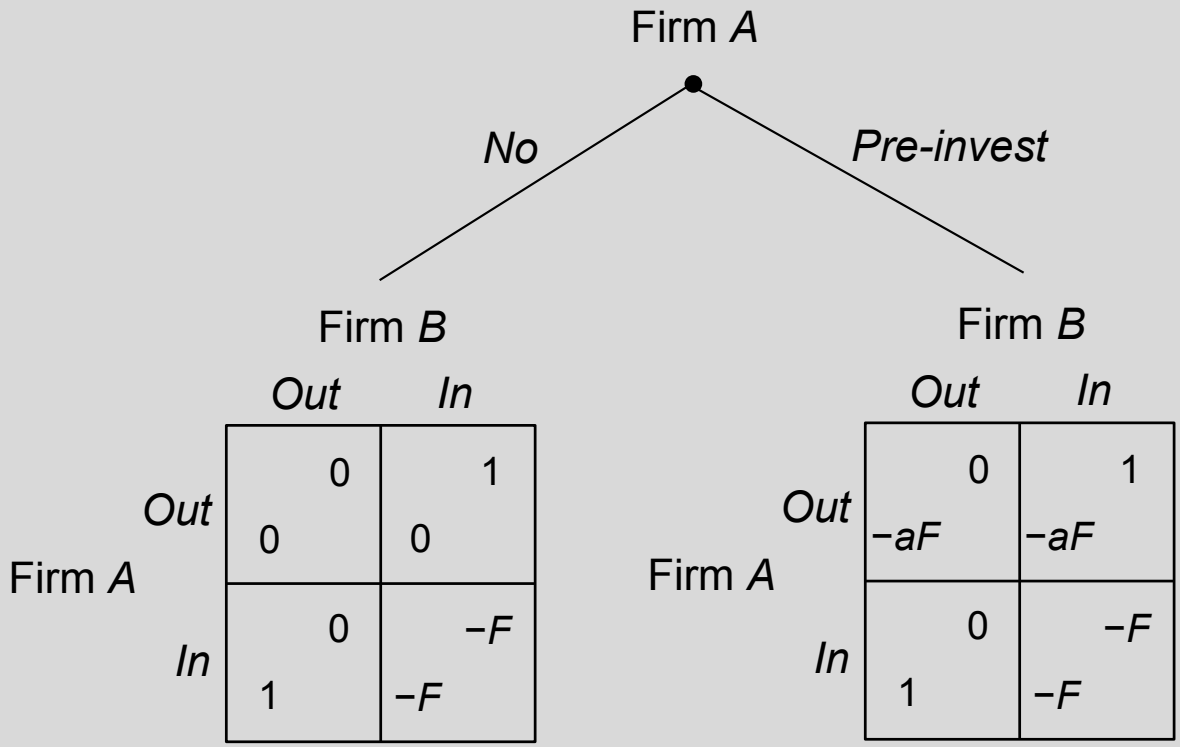


If Nature chooses the branch labeled with probability *p*, then Bob gets a payoff of 0 when Ann chooses *L* and he chooses *r*

If Nature chooses the branch labeled with probability 1 − *p*, then Bob gets a payoff of 3 when Ann chooses *L* and he chooses *r*

The key is that when Ann gets to move *L* or *R*, she does not know which are Bob's payoffs

Forward Induction



Firm *A* has the possibility of (irrecoverably) pre-investing a fraction *a* of the upfront investment *F* needed to enter the market

(Advanced topic we will not pursue here: What is a formal method of analysis that captures forward-induction reasoning?)